Modeling of Ribbed Slabs for Footfall Vibration Analysis

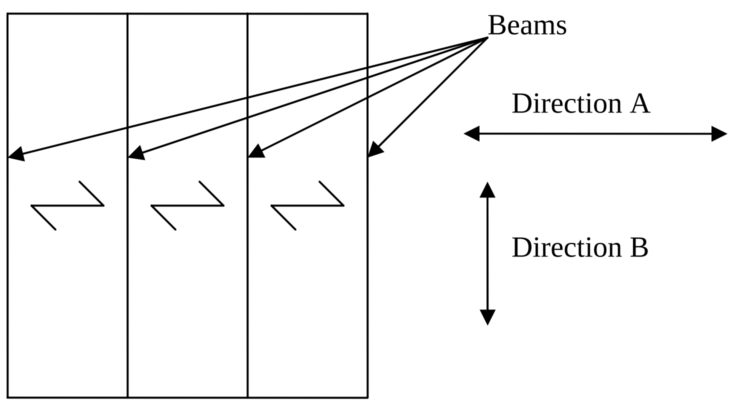
Baris Erkus[1], 15 May 2009

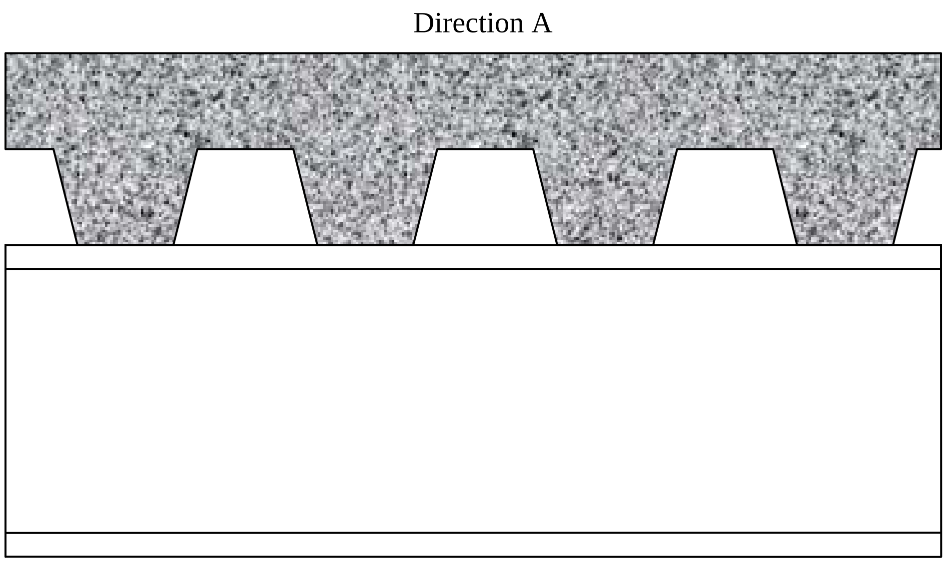
Introduction

This document provides a review of the computer modeling of ribbed composite slab stiffness and weight for the footfall vibration analysis based on the Steel Construction Institute (SCI) design guide (Smith et al., 2007).

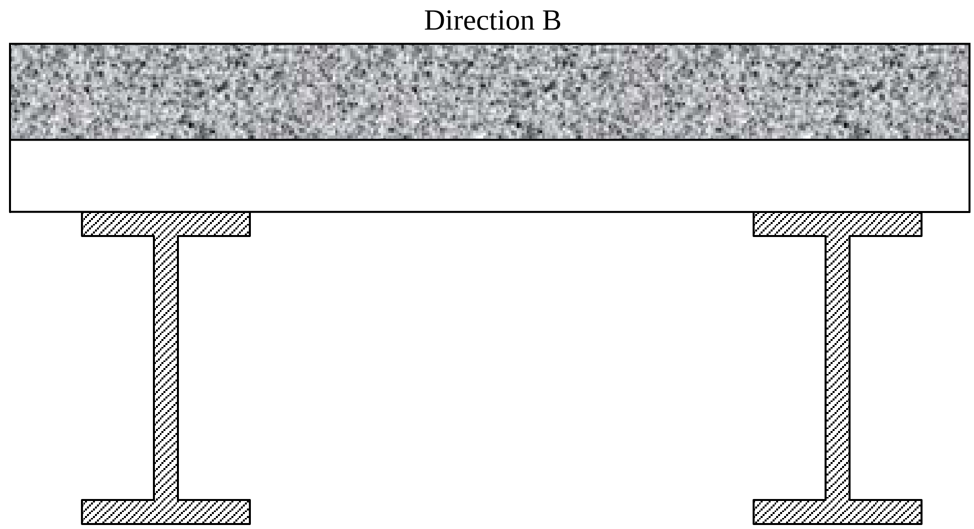
Problem Definition

Consider the following slab





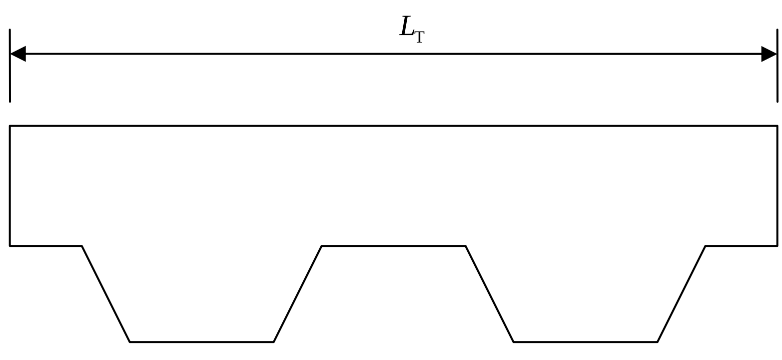
[1] Baris.Erkus@arup.com



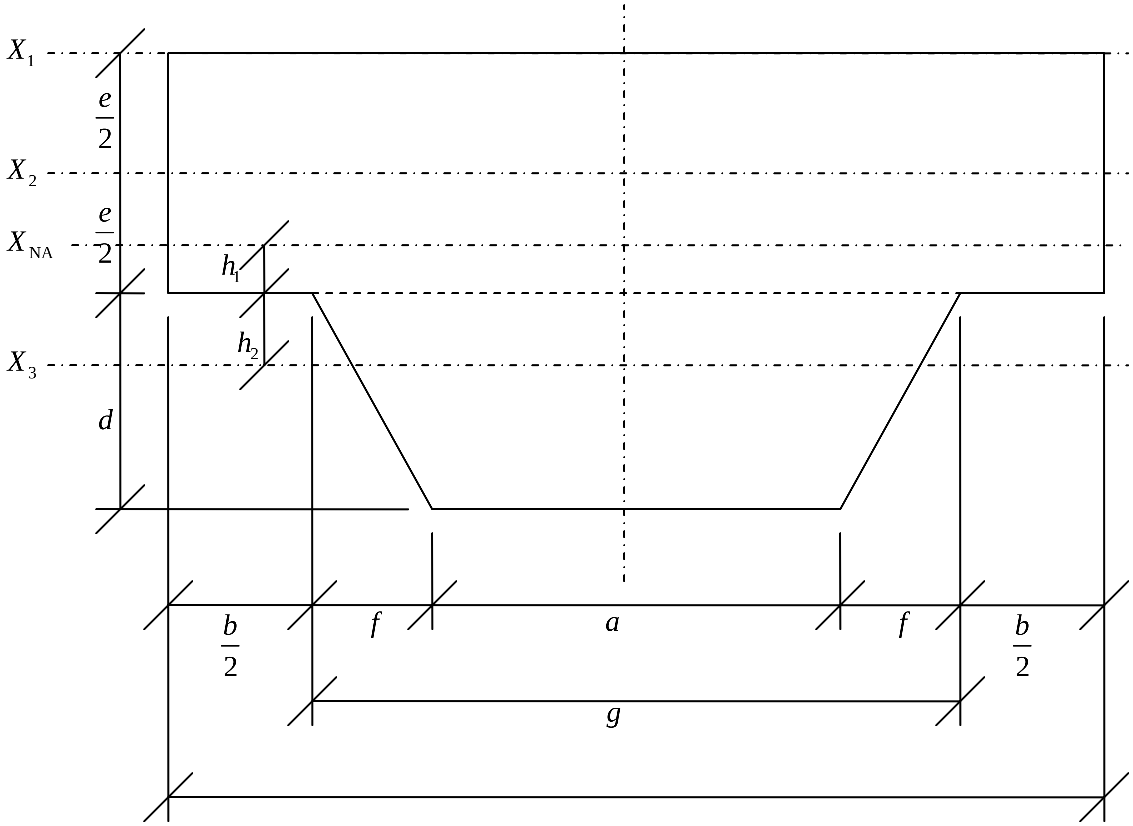
Our goal is to find an equivalent slab and beam system that can be modeled by finite elements analysis software. It is easily observed that the stiffness of the concrete filled deck will be different in both directions A and B, which is an orthotropic behavior. Some of the software can model orthotropic behavior in orthogonal directions such as ETABS and SAP2000, while other cannot such as GSA. We will consider both cases.

Equivalent Section Properties

Consider the ribbed slab dimensions shown below:



Let the number of the ribs be n. Now consider a unit element with single rib.



The properties of the top rectangular portion of the section are given below:

$$I_1 = \frac{1}{12}Le^3, \qquad A_1 = Le$$

The properties of the bottom trapezoidal portion are given below:

$I_2 = \dfrac{d^3(a^2+4ag+g^2)}{36(a+g)}$, and the neutral axis location $h_2 = \dfrac{d(2a+g)}{3(a+g)}$

$$A_2 = \frac{(g+a)}{2}d$$

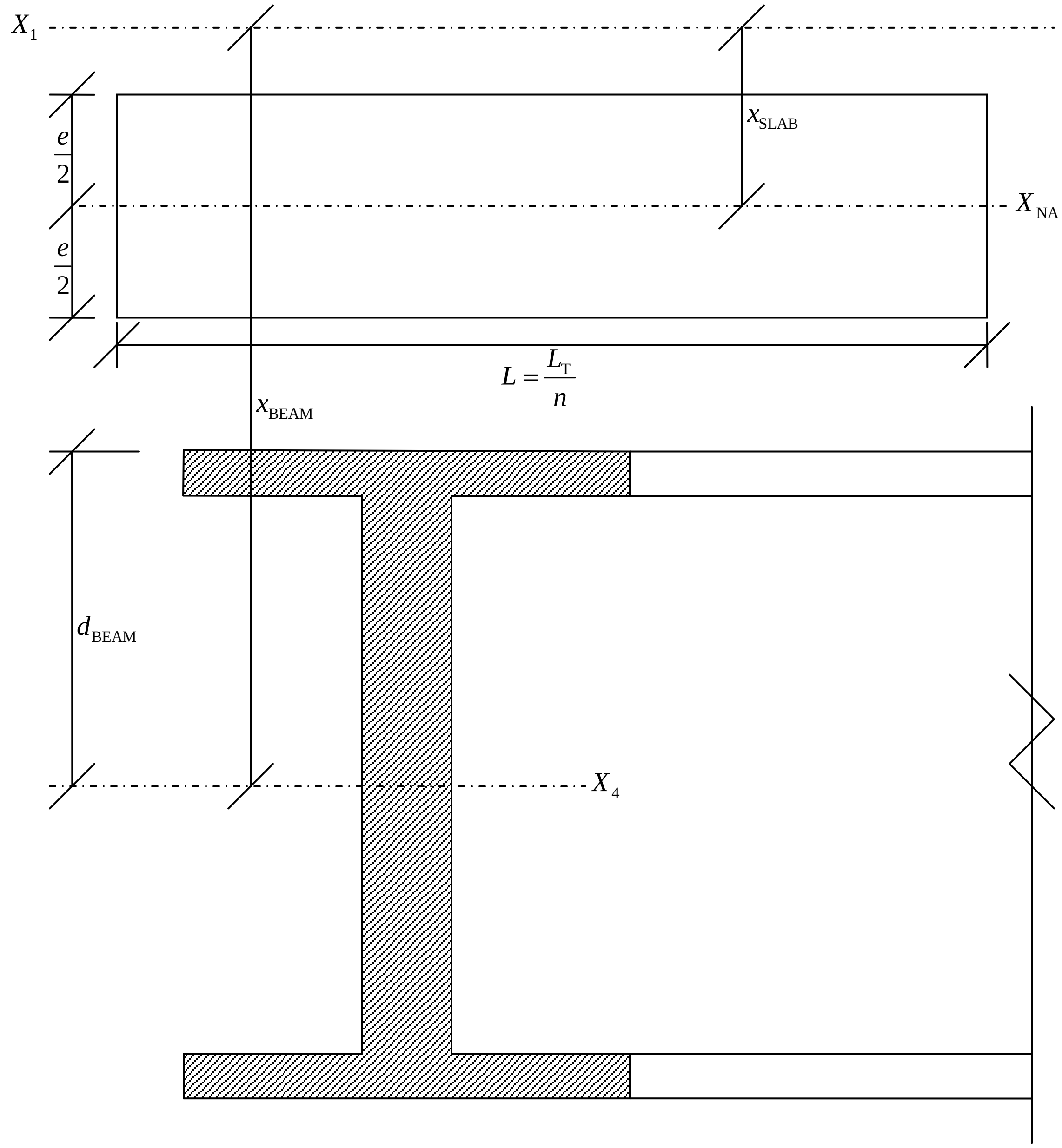
The neutral axis location of the ribbed section will be same as the center of area, i.e. taking moments of areas w.r.t. X_{NA} gives

$$A_1(\frac{e}{2}-h_1) = A_2(h_1+h_2), \qquad \Rightarrow \qquad h_1 = \frac{A_1\frac{e}{2}-A_2h_2}{(A_1+A_2)}$$

And the moment of inertia of the ribbed section w.r.t. the neutral axis is given by

$$I_{X_{NA}} = I_1 + A_1(\frac{e}{2}-h_1)^2 + I_2 + A_2(h_1+h_2)^2$$

Now, consider the equivalent section as follows:



Herein, x_{SLAB} and x_{BEAM} are the offset distances of the shell and beam elements from the finite element surface, which is generally taken as the top of structural slab. Note that X_4 is the beam neural axis.

$$x_{SLAB} = e - h_1$$
$$x_{BEAM} = e + d + d_{BEAM}$$

The stiffness properties of the top rectangular portion of the equivalent section have to be modified for equivalency in directions A and B separately. This can be done in several ways depending on the type of software. Some analysis packages allow modifying the modulus of elasticity, while others allow modifying the final stiffness in different directions. In both cases, the stiffness and the weight of the top rectangular concrete portion of the equivalent system in the Direction A have to be increased by a factor given by $I_{X_{\mathrm{NA}}} / I_1$. For example, if the modulus of elasticity is to be modified, than the new modulus of elasticity in the Direction A will be

$$E_{\mathrm{NEW}} = E_c \frac{I_{X_{\mathrm{NA}}}}{I_1}$$

Note that E is the dynamic modulus of elasticity of concrete which is suggested to be 38 kN/mm^2 for normal weight concrete and 22 kN/mm^2 for lightweight concrete (Smith et al., 2007).

In the Direction B, the stiffness should not be modified. Also, to have the same weight of the slab, the unit weight should be modified as

$$\gamma_{\mathrm{NEW}} = \gamma \frac{A_1 + A_2}{A_1}$$

The SCI design guide (Smith et al., 2007) also recommends using E_{NEW} in both directions if the analysis package does not have the option of providing orthotropic properties.

References

Smith AL, Hicks SJ and Devine PJ (2007) “Design of Floors for Vibration: A New Approach.” SCI Publication P354, The Steel Construction Institute, Berkshire, UK